Last week, we looked at some basic formulas related to Arithmetic Progressions. This week, we will look at a particular (and related) type of Arithmetic Progression — Consecutive Integers.

Look at the following three sequences:

S1 = 3, 4, 5, 6, 7

S2 = -1, 0, 1, 2, 3, 4, 5, 6, 7

S3 = 1, 2, 3, 4, 5, 6, 7, 8, 9

All of them are APs of consecutive integers so every formula we looked at last week is applicable here.

Sum of an AP = (n/2)[2a + (n-1)d]

Sum of the terms of S1 = (5/2)[2*3 + (5-1)1] = 25

Sum of the terms of S2 = (9/2)[2*(-1) + (9-1)1] = 27 (or treat it as a sum of 2, 3, 4, 5, 6, 7)

Sum of the terms of S3 = (9/2)[2*1 + (9-1)1] = 45

We will pay special attention to S3 i.e. an AP of consecutive integers starting from 1

Sum of the terms in this case = (n/2)[2*1 + (n-1)*1] (since a = 1 and d = 1)

(n/2)[2*1 + (n-1)*1] = (n/2)(n+1) = n*(n+1)/2

This is a formula we should be very comfortable with: sum of first n terms of a sequence of consecutive integers starting from 1 = n*(n+1)/2. It is very useful in a lot of situations. We can also derive a lot of other relations using this single formula.

For Example:

Example 1. What is the sum of positive consecutive even integers starting from 2?

- It is n(n+1) where n is the number of even integers (remember, n is not the last term; it is the number of total even integers).

- How?

- Say there are n numbers in the sequence and we want to find their sum: 2 + 4 + 6 + 8 + … We take 2 common out of them to get 2(1 + 2 + 3+ 4…). This is twice the sum of n consecutive integers starting from 1. So Sum = 2*n(n+1)/2 = n(n+1).

Example 2. What is the sum of positive consecutive odd integers starting from 1?

- It is n^2 where n is the number of odd integers (again, n is not the last term; it is the number of total odd integers).

- How?

- Say there are n numbers in the sequence and we want to find their sum: 1 + 3 + 5 + 7 + … Sum of 2n consecutive integers (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8…) will be 2n(2n+1)/2 and sum of n even consecutive integers (2 + 4 + 6 + 8 + …) starting from 2 will be n(n+1) so sum of the leftover n consecutive odd integers will be n(2n+1) – n(n+1) = n^2.

With this background, let's look at a question similar to an OG12 question:

Question 1: What is the sum of all the even integers between 99 and 401 ?

(A) 10,100
(B) 20,200
(C) 37,750
(D) 40,200
(E) 45,150

Solution: We can solve this question in multiple ways.

Required Sum = 100 + 102 + 104 + 106 + … + 400

Method 1:

We know the sum of consecutive even integers but only when they start from 2. So what do we do? We find the sum of even integers starting from 2 till 400 and subtract the sum of even integers starting from 2 till 98 from it! Note that we subtract even numbers till 98 because 100 is a part of our series.

How many even integers are there from 2 to 400? I hope you agree that we will have 200 even integers in this range (both inclusive)

Sum of these 200 integers = 200(201)

How many even integers are there from 2 to 98? Now we have 49 even integers here.

Sum of these 49 even integers = 49(50)

What is the sum of integers from 100 to 400? It will be 200(201) – 49(50) = 40200 – 2450 = 37750

Method 2:

100 + 102 + 104 + 106 + … + 400 = 2(50 + 51 + 52 + 53 + … + 200) (We take out 2 common and find the sum in brackets)

Sum in brackets = 50 + 51 + 52 + 53 + … + 200

We know the sum of consecutive integers but only when they start from 1. So we find the sum of first 200 numbers and subtract the sum of first 49 numbers from it. That will give us the sum of numbers from 50 to 200. Note that we subtract 49 numbers because 50 is a part of our series.

Sum of 1 + 2 + 3 + … + 200 = 200*201/2 = 20100
Sum of 1 + 2 + 3 + … + 49 = 49*50/2 = 1225 (I am doing these calculations here only for clarity. Normally, I would like to carry all these till the last step, then take common, divide by whatever I can etc so that I have very few actual calculations left.)

Therefore, 50 + 51 + 52 + 53 + … + 200 = 20100 – 1225 = 18875

Then 100 + 102 + 104 + 106 + … + 400 = 2*18875 = 37750

Answer (C)

I hope you see that questions on arithmetic progressions are generally quite simple. Next week, we will move on to geometric progressions. Till then, keep practicing!